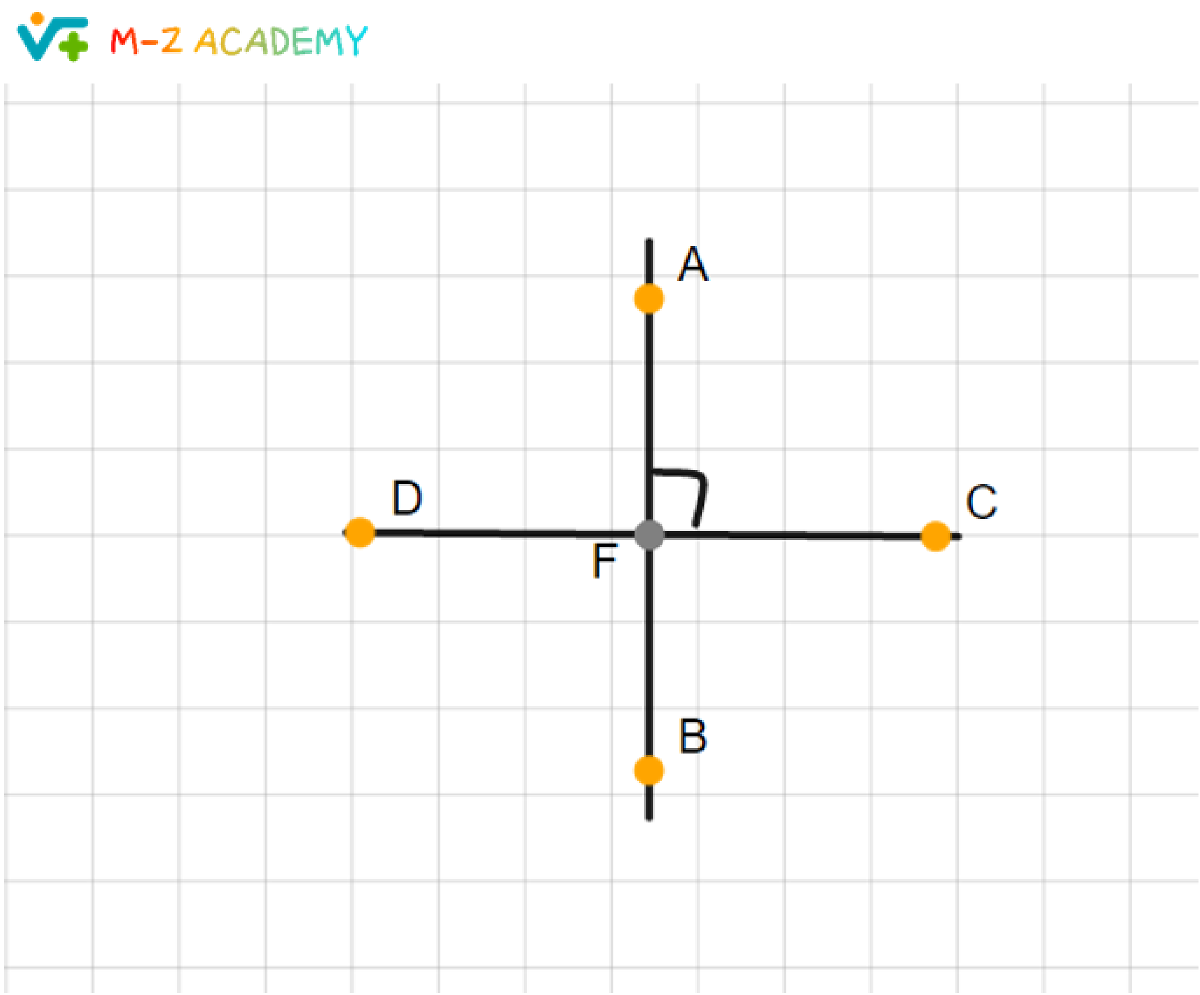
In this post, we will explore fundamental geometry concepts such as the perpendicular bisector and angle bisector. These are the following topics we will study in this article:

1. What is a perpendicular bisector?
2. The perpendicular bisector theorem
3. What is an angle bisector?
4. The angle bisector theorem
5. Examples of both angle bisector and perpendicular bisector

What is a perpendicular bisector?

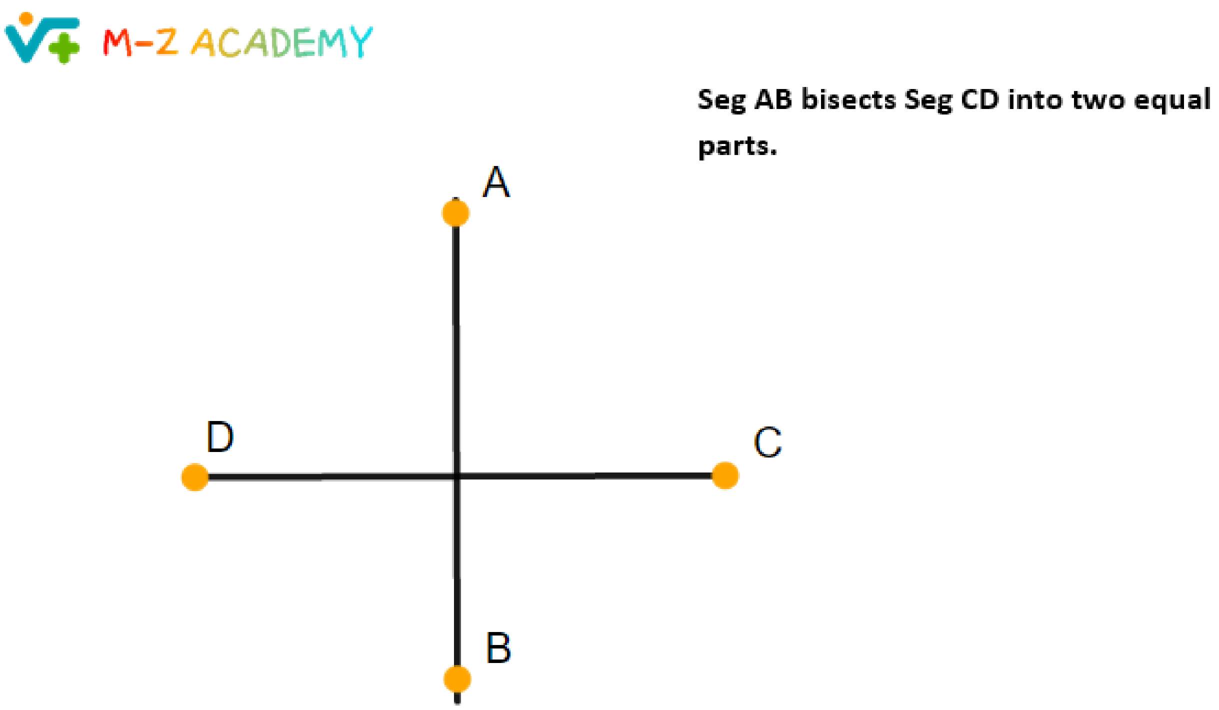
A perpendicular



Let us gradually break down the perpendicular bisector by first defining what perpendicular is. If two distinct lines, rays, or line segments intersect at 90° or form a right angle with each other, it is called perpendicular lines.

The above figure shows that the line segment AB intersects the line segment CD at point F, thus forming a right angle. Hence, they're called perpendicular lines.

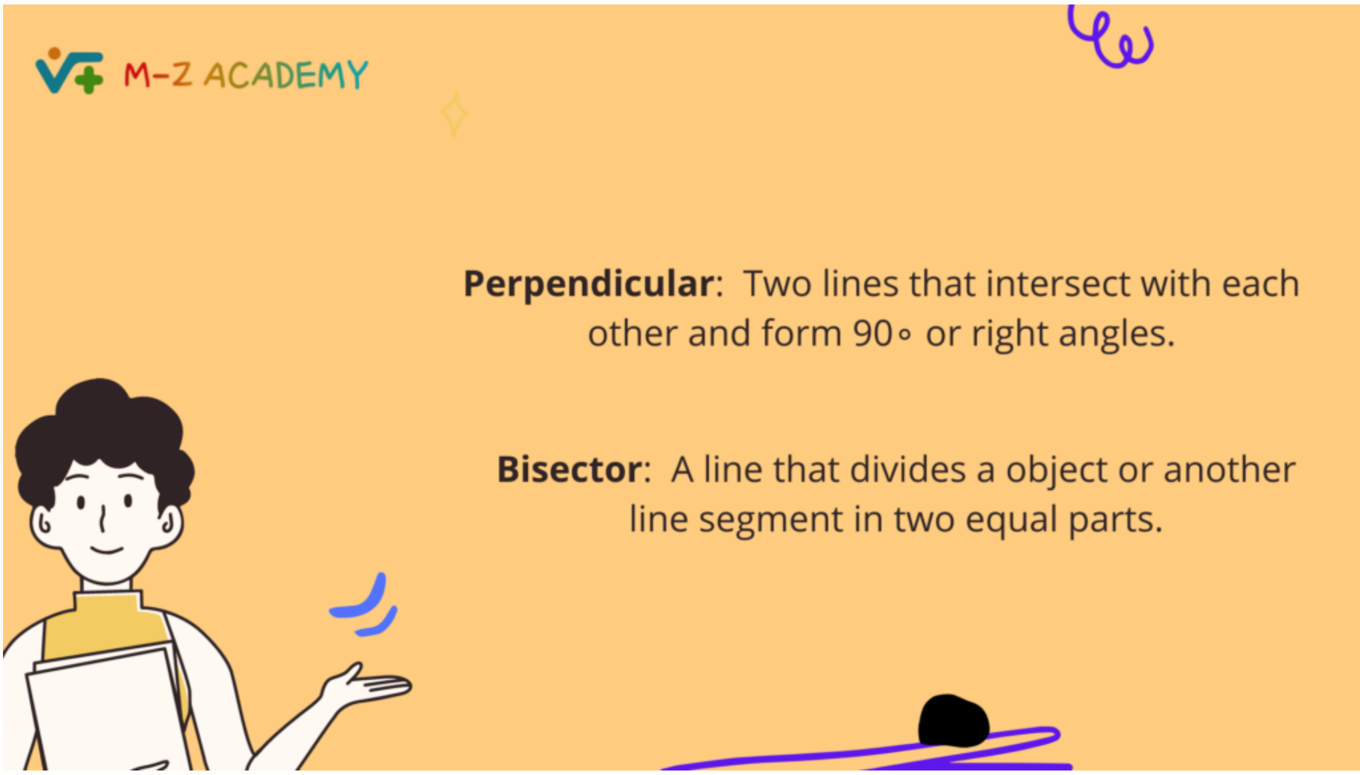
A bisector



A bisector is an object (line, line segment, or ray) that intersects another object or line segment in such a way that the segment is divided into two equal parts. Also, a bisector cannot bisect a Line as Line does not have a finite length.

Let's take a look at the example above to understand how a bisector works. In the above figure, seg AB bisects seg CD such that it divides the segment into two equal parts.

A perpendicular bisector

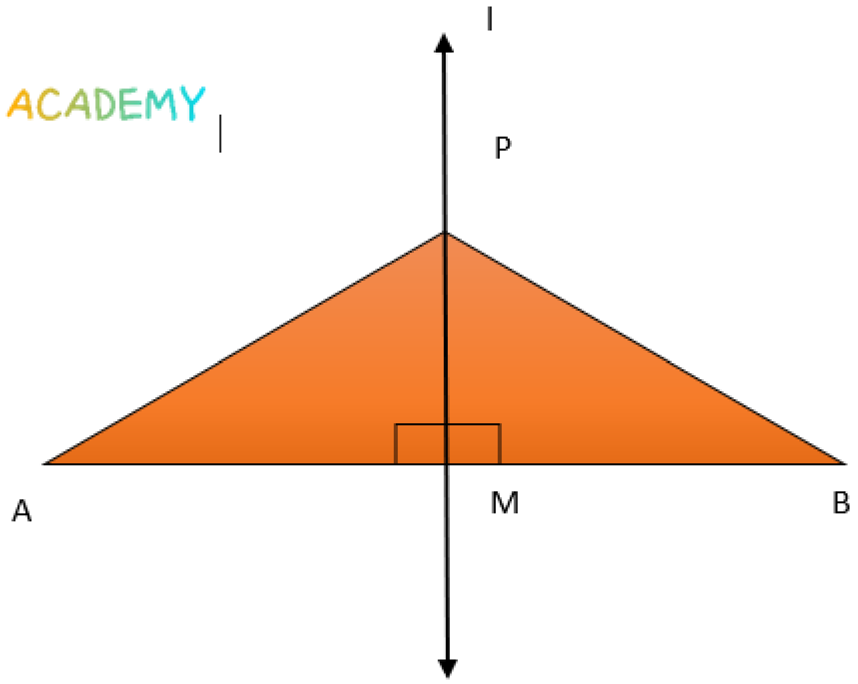


Once we understand what a perpendicular line and bisector are, defining a perpendicular bisector becomes simple.

A perpendicular bisector is a line, line segment or ray that bisects a segment at a right angle and divides the segment into two equal parts. In short, a perpendicular bisector is a combination of a perpendicular line and a bisector.

Further to know how to construct a perpendicular bisector, I recommend you watch this following video

Perpendicular bisector theorem



Furthermore, by combining all these points, we may finally comprehend the perpendicular bisector theorem.

Statement: Every point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.

Given: line l is the perpendicular bisector of seg AB at point M. Point P is any point on l.

To prove: PA = PB

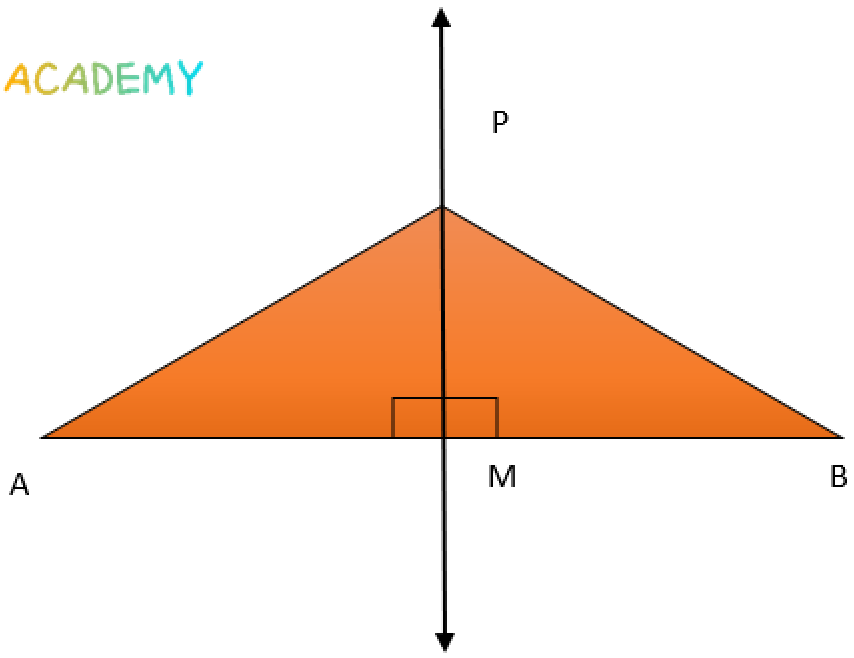
Construction: Draw Seg AP and Seg BP

Proof:	In Δ PMA and Δ PMB
Seg PM ≅ Seg PM	Common side
∠PMA ≅ ∠PMB	Each is a right angle
Seg AM ≅ Seg BM	Given angle
∴ Δ PMA ≅ Δ PMB	SAS test (side angle side test)

∴ Seg PA ≅ Seg PB
∴ l (PA) = l (PB)

Hence every point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.

Converse of perpendicular bisector theorem



Statement: Any point equidistant from the end points of a segment lies on the perpendicular bisector of the segment.

Given: Point P is any point equidistant from the end points of seg AB. That is, PA = PB.

To prove: Point P is on the perpendicular bisector of seg AB.

Construction: Take mid-point M of seg AB and draw line PM.

Proof: In Δ PAM and Δ PBM

seg PA seg PB	perpendicular bisector theorem
seg AM seg BM	midpoint
seg PM seg PM	common side

 Δ PAM Δ PBM

 ∠ PMA ∠ PMB

But ∠ PMA + ∠PMB = 180°

∠ PMA + ∠PMA = 180°

2∠ PMA = 180°

 ∠ PMA = 90°

 seg PM ⊥ seg AB

But Point M is the midpoint of seg AB according to construction

Therefore, line PM is the perpendicular bisector of seg AB. So, point P is on the perpendicular bisector of seg AB.

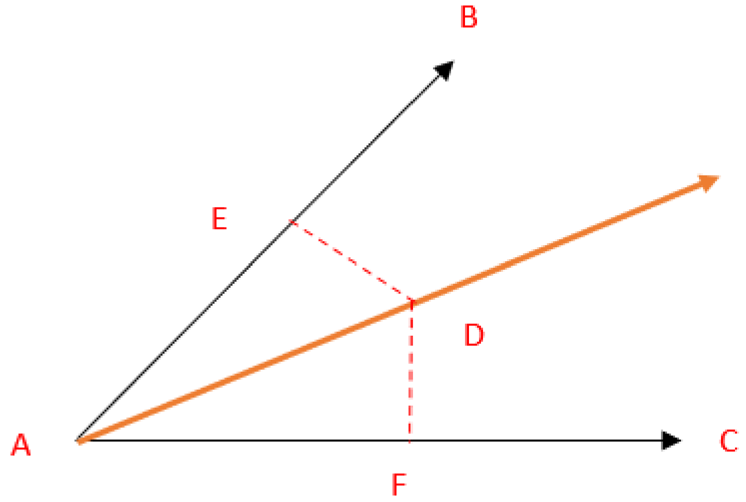
What is an Angle bisector?

Just like how a bisector divides a line segment into two equal halves, an angle bisector is a ray, line, or line segment that divides an angle into two equal parts.

Construction of an angle bisector

Please refer to the below video to visualize the construction of an angle bisector.

Angle bisector theorem



Statement: If a point is on the angle bisector, then it is equidistance from the sides of the angle.

Given: Ray A is the bisector of ∠BAC
Point D is any point on Ray A.

To find: Seg ED Seg DF

Solution:

We know that Ray A bisects ∠BAC

 Ray AB ⊥ Seg ED and Ray AC ⊥ Seg DF

According to the figure,

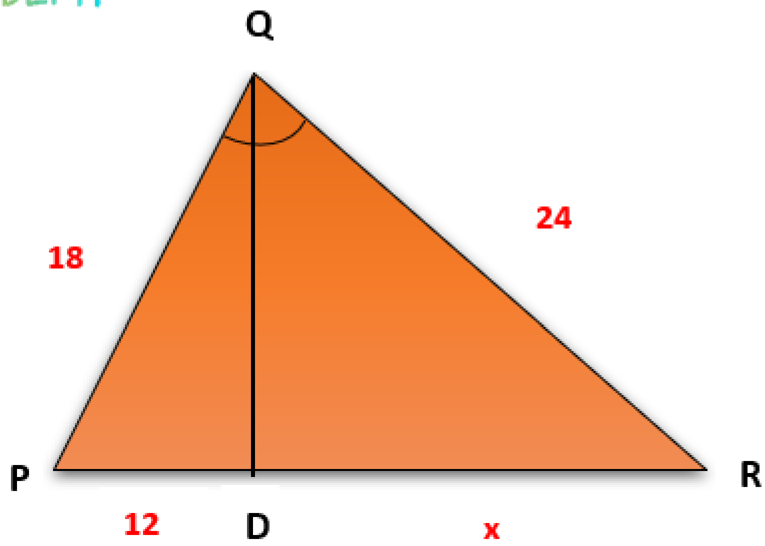
∠ AED and ∠ AFD are right angles all perpendicular lines are right angles
 ∠ AED ∠ AFD all right angles are congruent

∠ BAD ∠ FAD	definition of angle bisector
AD AD	common side
Δ AED Δ AFD	AAS
Seg ED Seg DF	corresponding parts of congruent triangle is also congruent

Hence, it is proved that if a point on the angle bisector (e.g., D), then it is equidistant from the side of the angles (seg ED seg DF).

Solved Examples:

1) Find the value of x for the given triangle using the angle bisector theorem.



Solution:

Given that,

PD = 12 PQ = 18 QR = 24 DR = x

According to angle bisector theorem,

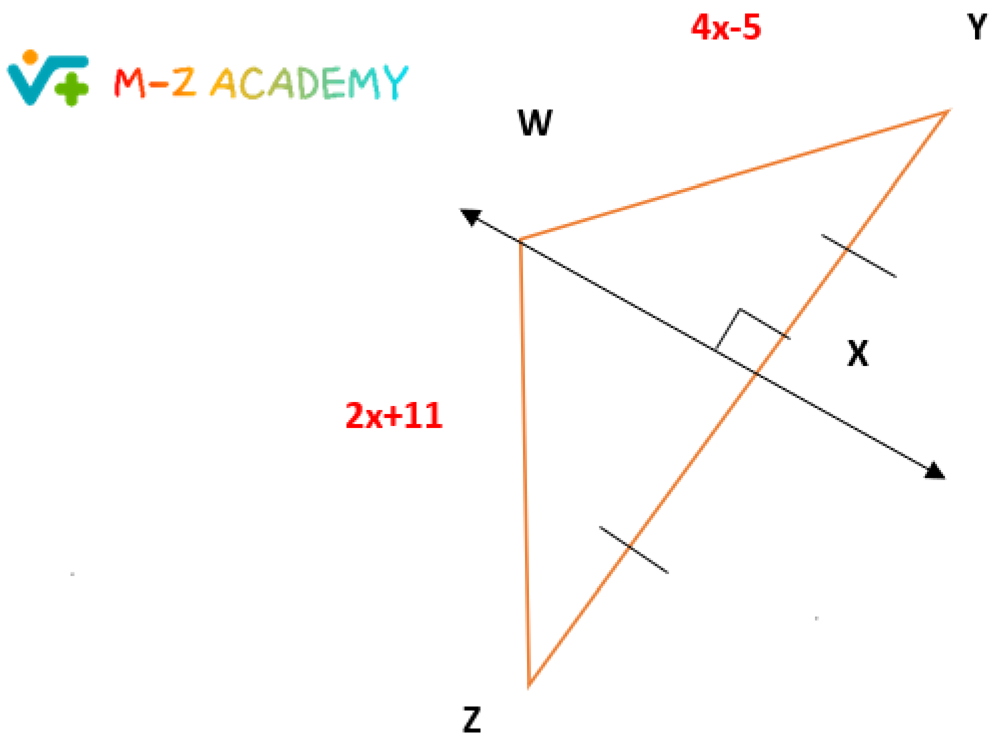
PD/PQ = DR/QR

Now substitute the values, we get

12/18 = x/24
X = (⅔)24
x = 2(8)
x = 16

Hence, the value of x is 16.

2) Find x and length of each segment.



Solution:

In the above figure, the line WX is perpendicular bisector to segment ZY.

 ∠ WXY= 90° By perpendicular bisector theorem
 ZX = XY

Also,

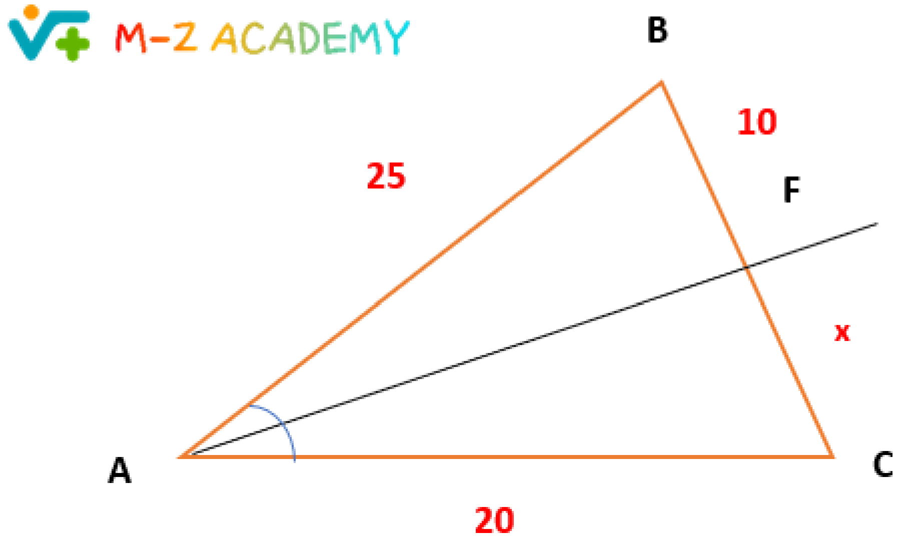
Seg WZ Seg WY By perpendicular bisector theorem
 2x + 11 = 4x – 5 Given
 16 = 2x
 X = 16/2
 X = 8

Length of segments

Seg WZ = 2x + 11
= 2(8) + 11
= 16 + 11
= 27
Seg WY = 4x – 5
= 4(8) – 5
= 32 – 5
= 27

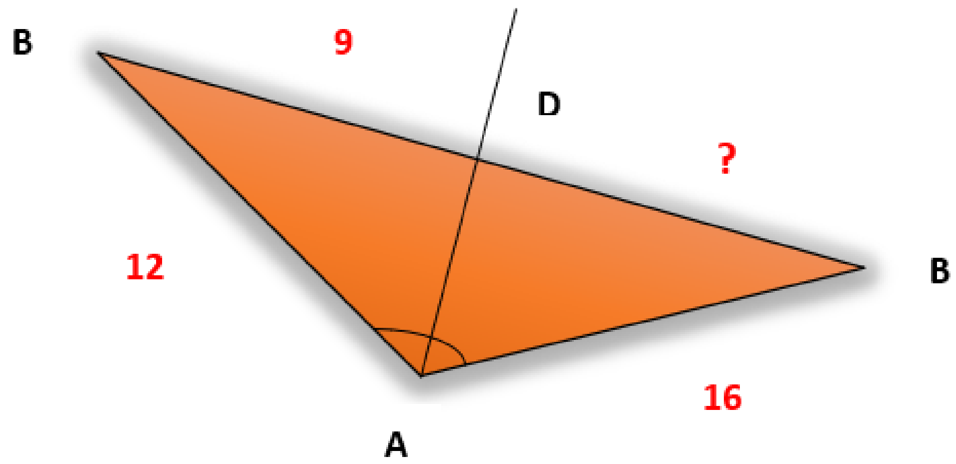
Unsolved Examples:

1) Find the value of x in Δ ABC.



Ans: 8

2) In Δ ABC pictured below, AD is the angle bisector of ∠ A. If CD = 9, CA = 12 and AB = 16, find BD.



Ans : BD= 12

Related topics:

- Isosceles triangle theorem
- Basic Concepts in Geometry
- Pythagorean Theorem

If you have any doubts regarding the article or the examples, please post them in the comments section.